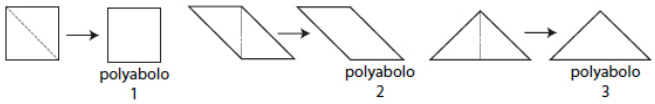
MathCounts Handbook 2012-13 Problem 48 (p. 13)
Detailed Solution

48. _____ poly-aboloes A polyabolo is a polygon formed by joining congruent isosceles right triangles in such a way that each triangle shares a side with at least one other triangle. Three distinct polyaboloes can be formed from two triangles, as shown below. If rotations and reflections are not counted separately, how many distinct polyaboloes can be formed from three triangles?



Math Background: **Polyabolo** (pronounced \pä-lē-'ä-bo-lo\; also known as **polytan**) is generally considered part of the "recreational math," thus not specifically covered in regular school math. However, isosceles right triangle and transformations (slide, turn, and flip, or in more formal and MathCounts Vocabulary terms respectively, translation, rotation, and reflection) are concepts taught in Upper (Grades 4-6) Elementary Math and Pre-Algebra. This problem is designated at "difficulty level 3" (on a scale of 7), which means appropriate for students starting in Math 6 or Pre-Algebra.

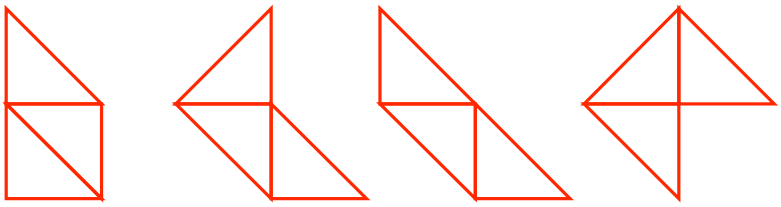
A single isosceles right triangle is a **monabolo**. Since "rotations and reflections are *not* counted separately," there is only one way and one distinct polyabolo in this case.

Two identical and congruent isosceles right triangles can form **diaboloes**. The diagrams below the problem demonstrate three distinct ways of arranging the triangles.

The challenge to us is to arrange three identical and congruent isosceles right triangles and to find how many possible ways to form distinct **triaboloes**.

Be mindful of "equivalent" triaboloes (same outer boundaries in addition to rotations and reflections), for instance (not so exhaustive):

∴ The answer to the problem is **4** (polyaboloes).



Extension: How many distinct polyaboloes can be formed from four triangles?

(Answer: You can arrange 4 congruent isosceles right triangles into 14 distinct **tetraboloes**.)